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Is Scientific Knowledge an Inexhaustible Economic Resource?

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Abstract

Some comments on R. S. Percival, 'The Metaphysics of Scarcity: Popper's World 3 and the Theory of Finite Resources' (Percival 1996). Section 1 introduces and explains the central problem. Section 2 investigates in some detail the logical theorem presented by Percival in §45, on which much of his argument depends, and questions its significance and applicability. Section 3 offers an alternative explanation of the possibility of unlimited technological progress.

Contents

1 Introduction

(1) Ray Percival has asked me to say something in public about his paper 'The Metaphysics of Scarcity' (Percival 1996).[1] I do so a shade reluctantly, since his topic is not one that I have ever thought deeply about; but also gladly, since it provides an opportunity to look with care at the subtly misleading logical argument on which he relies. His paper is written in numbered paragraphs, and it is to these that I shall refer in my discussion.

(2) Although there are unclarities, Percival's problem appears to be *the problem of explaining how indefinite economic growth is possible, given that most natural resources are, or will become, scarce.* By no means does he think that 'continued growth in economically useful inventions' (§26) is inevitable, let alone something that can be scientifically predicted; yet he anticipates such growth, and wishes to explain it.

(3) Following Simon (1981), Percival dismisses the view that natural resources are 'simply portions of matter/energy just waiting to be discovered' and maintains that all resources 'are created by an interaction between the human mind, theories and the physical world' (§2). This is a perplexing overstatement as far as air, water, sunshine, and other necessities of life are concerned; for though, in response to chemical pollution, the water that we now drink in industrialized countries and—to a lesser extent—the air that we breathe are heavily contaminated with theory, it was not always so. Yet it may be admitted that animal (and even vegetable) inventiveness, most of it unconscious, can endow previously useless material substances with vital economic significance. This is one aspect of 'active Darwinism' (Popper 1992, pp. vii–ix). But since there is competition within species, not only between species, the ability to colonize new niches does not palliate in the least one aspect of 'passive Darwinism', the brutal fact that living organisms are obliged to share a limited budget of physical resources, and that those less lucky in the distribution are eliminated.

(4) Popper often stressed that the possession of objective knowledge, being largely an exercise in exosomatic adaptation, gives mankind a distinct advantage over other species in this struggle: we can sacrifice our theories instead of our skins (Popper 1972, p. 122). Percival's thesis, if I understand him, is that there is a second way in which objective knowledge can relieve the rigours of natural selection. For theoretical knowledge also gives us the means to exploit physical resources in indefinitely many ways, and with increasing efficiency, so that there need be no upper limit to the benefit extractable from a finite bundle of sufficiently varied goods. Land is cited

[1] I thank Percival for useful criticisms of an earlier draft of the present set of comments.

as an exception, for until we give way to those 'highly sophisticated, self-reproducing computers' who are doomed to be our descendants (§61), we all need an irreducible volume of space in which to live. But the suggestion is that, in general, the unconfined power of our intellectual resources may more than compensate for physical shortages.

(5) The centre of Percival's argument is a small theorem to the effect that the content of a theory, especially a scientific theory, is actually infinite (§§44-46), and that what we can know of it is potentially infinite. He endorses the conclusion drawn from this theorem both by Popper (1974, section 7), and by Bartley (1990, p. 33) that 'our best *existing* knowledge is *unfathomed and unfathomable*'. The suggestion is that this inexhaustibility of our objective knowledge can explain the infinite versatility of our invention. Percival writes for instance (§28):

> A theory ... can be applied an infinite number of times and in an infinite number of different useful projects because of its universal reference to all space and time and because of its infinitely varied logical and (in the case of scientific theories) information content.

(6) In the next section, an excursion into technical logic, I shall investigate the scope and applicability of the theorem that Percival cites. Those who have no taste for barren subtlety may proceed to section **3**. There I contest Percival's explanation of the limitlessness of technological progress, and replace it with an explanation that equally makes appeal to the unfettered power of our intellectual resources but, I think, more accurately locates them. I shall maintain, in brief, that it is not because our theories are so strong, but because they are not so strong, that there is so much room available for so much inventive application.

2 Some Points of Logic

2.1 Terminology & Notation

(7) Throughout this section I shall make use of a number of items of standard logical notation, together with some ideas that are less familiar. The signs $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\forall$ are respectively the signs of *negation* [**not**], *conjunction* [**and**], *disjunction* [**or**], *conditional* [**if ... then**], *universal quantification* [**for all**]. The symbol '$\vdash$' [**yields** or, in the present context, **implies** or **entails**] stands for *derivability* or *deducibility* in the sense of classical elementary logic; the symbol '$\equiv$' [**is logically equivalent to**] stands for mutual derivability. The connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$ are understood to be operators, in the first instance, on syntactical items called *statements*. (It might be better to call them sentences, in order to emphasize their formal or morphological character, but it is less confusing if Popper's usage is followed.) Two statements that are not identical are *distinct*. Two statements that are not logically equivalent are *logically distinct*. Logical truths—statements that are derivable from all statements, or from none—will, laxly but conveniently, be called *tautologies*. There are infinitely many distinct tautologies, but of course no two are logically distinct.

(8) If X is a set of statements and $X \vdash x$, we shall say that x is a [*logical*] *consequence* of X. The set of consequences of X is written $\mathsf{Cn}(X)$. A [*deductive*] *theory* $\mathbf{X}$ is a set of statements closed under the operation Cn; that is to say, $\mathbf{X} = \mathsf{Cn}(\mathbf{X})$. A theory is *inconsistent* if it contains every statement; otherwise *consistent*. The connectives $\wedge$ and $\vee$ and the relation $\vdash$ may be extended in a natural way from statements to theories: $\mathbf{Z} \vdash \mathbf{X}$ if & only if $\mathbf{X} \subseteq \mathbf{Z}$. Whatever the terminology may suggest, it is of no importance to this paper whether the consequence operation Cn is characterized syntactically (by rules of derivation) or semantically (in terms of models).

(9) If $\mathbf{X} = \mathsf{Cn}(\{x\})$ for some statement x, we call $\mathbf{X}$ [*finitely*] *axiomatizable*. If $\mathbf{X}$ is consistent, and has no consistent proper extension $\mathbf{Z}$, then $\mathbf{X}$ is *maximal* (or *complete*). If $\mathsf{Cn}(\{x\})$ is both axiomatizable and maximal, $\mathsf{Cn}(\{\neg x\})$ has no proper consequences but tautologies (for if $\neg x \vdash z$ then $\neg z \vdash x$). In this case the theory $\mathsf{Cn}(\{\neg x\})$, and by extension the statement $\neg x$, are called *irreducible* (Tarski 1936, section 4). It follows at once that $\mathbf{X}$ has only finitely many distinct consequences if & only if $\mathbf{X} = \mathsf{Cn}(\{z_0, \ldots z_{k-1}\})$ where each of the z_i is irreducible. It is plain that $\mathbf{X}$ cannot have a finite content unless it is axiomatizable, and thus identical to $\mathsf{Cn}(\{x\})$ for some statement x.

2.2 A Small Theorem

(10) The proof that Percival gives (§45) of the infinitude of the content of any interesting axiomatizable theory t comes straight from note 18 of (Popper 1974, Popper 1976). Percival writes:

> Suppose an infinite list of statements that are pair-wise contradictory and which individually do not entail t: $a, b, c, \ldots$ Then the statement 't or a or both' follows from t. The same holds for each and every one of the statements in the infinite list. Since the statements in the list are pair-wise contradictory one can infer that none of the statements 't or a or both', 't or b or both' etc., is interderivable. Thus the logical content of t must be infinite.

(11) More explicitly: if $t \vee a \vdash t \vee b$ then $a \wedge \neg b \vdash t$. But by hypothesis $a \wedge b \vdash t$. Hence $a \vdash t$, contrary to supposition. We may conclude that no element of the list $t \vee a, t \vee b, \ldots$ implies any other, and hence that no two are logically equivalent.

2.3 A Small Generalization

(12) This result is rather general, since the requirement that there exist a denumerable sequence $a, b, c, \ldots$ of statements so related to t is not a severe one. Popper himself notes that '[f]or most t's, something like a: "the number of planets is 0", b: "the number of planets is 1", and so on, would be adequate' (1974, p. 19; 1976, pp. 26f). The proof does not even require that each pair of elements of the sequence consist of contraries; only that each pair entail t. I do not know whether either of these conditions is necessary for t to have infinitely many logically distinct consequences, but there are closely related conditions that are separately both necessary and sufficient for this: (i) there exists a denumerable sequence $\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \ldots$ of pairwise incompatible theories that individually do not imply t; (ii) there exists a denumerable sequence $\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \ldots$ of theories that pairwise, but not individually, imply t. Since (i) is stronger than (ii), we need prove only the sufficiency of (ii) and the necessity of (i). These results are of only supplementary significance in the present context, and those who are not itching to see proofs of them are invited to resume the main line of argument in **2.4**.

(13) The sufficiency of (ii) is established in much the same way as in **2.2**. Instead of constructing a sequence of statements $t \vee a, t \vee b, \ldots$ we construct a sequence of theories $t \vee \mathbf{X}, t \vee \mathbf{Y}, \ldots$. (The disjunction of two theories consists of their common consequences.) Now if any one of these new theories is not axiomatizable, then t must have infinite content; for if t had only finite content, then its subtheories would have finite content too. In other words, we may assume that each of $t \vee \mathbf{X}, t \vee \mathbf{Y}, \ldots$ can be axiomatized by a single statement, and proceed as before.

(14) The proof of the necessity of (i) is a little trickier, and can only be sketched. First recall from **2.1** that t has only finitely many logically distinct consequences if & only if it is logically equivalent to the conjunction of a finite set of irreducible statements. It follows, not quite obviously, that t has finite content if & only if $\neg t$ has only finitely many maximal extensions, all axiomatizable. In other words, it is necessary & sufficient for t to have infinite content that $\neg t$ has either infinitely many axiomatizable maximal extensions, or at least one unaxiomatizable maximal extension. Now it is immediate from a theorem of Mostowski (1937, Theorem 8, p. 13; reported on p. 370 of Tarski 1936) that if a theory has even one unaxiomatizable maximal extension, then it has infinitely many maximal extensions. We may conclude that if t has infinite content, then $\neg t$ has infinitely many maximal extensions. From this set of maximal extensions we may extract a denumerable sequence $\{\Omega_i \mid i \in \mathcal{N}\}$ of theories individually implying $\neg t$. Since each such Ω_i is consistent, none of them implies t, and since each is maximal, they are pairwise incompatible. In this way (i) is proved.

2.4 Informative Content

(15) Popper admits that the result of **2.2** is both 'well known' and 'trivial' (though he means trite), but suggests that it may appear rather more significant if it is phrased not in terms of logical content, but in terms of the closely related idea of informative content. He calls *the informative content of a theory* the class of statements that are incompatible with it; the class, that is, of statements that it excludes or forbids (Popper 1974, p. 18; 1976, p. 26). Now on the one hand it is quite plain that, since $\mathbf{X}$ implies x if and only if $\mathbf{X}$ is incompatible with $\neg x$, the informative content of a theory is infinite whenever its logical content is infinite. But on the other hand we can see that amongst the elements of the informative content of a theory will be many statements that the inventor of the theory may not have had in mind when he formulated it, and may indeed never come to appreciate. Kepler's theory, to vary slightly the example given in turn by Popper, by Bartley, and by Percival, excludes Newton's theory, at least in the presence of the assumption that the individual planets have non-zero mass. Kepler's first law, which says that the sun occupies one focus of the elliptical orbit of each planet, is corrected by Newton's theory in at least two ways: first, by introducing perturbations due to gravitational attraction among the planets; and second by insisting that, even in a one-planet system, it is not the sun itself but the centre of mass of the sun-planet complex that is at the focus of the elliptical orbit. The informative content of a theory, that is, is in some sense not present in its entirety to the inventor of the theory, and exactly the same therefore applies in the case of its logical content.

(16) Later in the section (in **2.11**) and throughout section **3**, I shall have cause to applaud this idea that the content of a theory is determined by what it rules out—an idea that goes back, as Percival notes, to the discussion of empirical content in §§31–35 of *Logik der Forschung* (Popper 1934). For the time being, if I may, I shall continue to investigate the significance of the theorem stated and proved in **2.2**.

2.5 Unfathomed Knowledge

(17) Popper describes the situation in the words: '*we never know what we are talking about*' (1974, p. 19; 1976, p. 27). Bartley (1990) says: '*we do not know what we are saying or … what we are doing*'. The point in each case is that if understanding a theory to the full requires understanding all its logical consequences, we cannot be said to understand to the full even our own creations. A similar evaluation was given by Ryle in his inaugural lecture at Oxford (1945, p. 7; 1971, p. 198): 'Thus people can correctly be said to have only a partial grasp of most of the propositions they consider. They could usually be taken by surprise by certain of the remoter logical connexions of their most ordinary propositions.' As we shall see in **2.6**, Ryle went on to qualify this judgement in an important way.

(18) Now I will admit that I am not averse to the main line of thought here, especially not to the idea that we often discover in our theories consequences that we never suspected. The history of remarkable theorems in Euclidean geometry, one of the most extensively studied of all mathematical theories, is evidence enough that our knowledge has an uncanny ability to surprise us; I need only cite my favourite theorem in Euclidean geometry, Morley's theorem (which says that the points of intersection of adjacent trisectors of adjacent angles of a triangle always form an equilateral triangle), or some of the theorems collected in (Evelyn, Money-Coutts, & Tyrrell 1974). But though sympathetic to the general idea that we do not know half of what we know, I think that a few unsympathetic comments deserve a hearing.

2.6 Some Difficulties with this Interpretation

(19) In the first place, note that the interpretation given to the result is dangerously strong. For since most of, or even all, the non-tautological consequences of a theory themselves have infinitely many logically distinct consequences—all those, in fact, that are not equivalent to the conjunction of some finite number of irreducible statements—we are forced to acknowledge that we do not understand properly most, or even any, of the non-tautological consequences of any theory that we hold. If understanding its consequences is what is important to understanding a theory, then we do not really understand theories at all; it is not just that our understanding is limited—it is unbegotten. I for one therefore want to hold on to the alternative idea, also endorsed by Popper (and by many others, such as Collingwood), that the real path to understanding a theory is by way of understanding its response to the problem situation that provoked it. Bartley (1990, p. 34) lumps together these different varieties of understanding, but they deserve to be kept cleanly apart.

(20) The second point to be made is that Popper and Bartley can hardly be drawing attention only to the frailty of our subjective apprehension of the items of objective knowledge that we (and others) have constructed. There must be more to what they are insisting on than that we are not logically omniscient, that we are unable to recognise all the consequences of what we say. If that were really all that was meant, then it would be hard to see why the proof should bother to establish the (admittedly simple) point that under suitable conditions a theory t has infinitely many consequences that are logically distinct from each other. For it is even more straightforward to establish that every theory, even one that states only a tautology, has infinitely many syntactically distinct consequences; that is, that there are infinitely many distinct statements that are derivable from any logical truth. Everyone who has studied logic knows (though may not be able to prove) that there exist infinitely many syntactically distinct tautologies, indeed infinitely many distinct forms of tautologies. It would be plainly an exaggerated idealization to suggest that any logician, however skilled, could recognise all these distinct tautologies. But we ought to jib at the idea that the understanding of $y \to y$ requires any ability to recognise all its equivalents in propositional logic (let alone all its equivalents in elementary predicate logic) as logical equivalents. Indeed, no one actually has the psychological prowess (or time) to recognise all tautologies of the simplest form $y \to y$. Once we reach instances of this scheme involving several million variables and many billions of pairs of parentheses we are beyond what is accessible even to the keenest brain. And it cannot be thought that Popper and Bartley imagine that there is a sharp and significant difference here between the transparent relation of logical equivalence and the opaque relation of logical implication. The mere possession of infinitely many equivalents, or of infinitely many consequences, though having psychological implications of the most banal kind, does not in itself make a theory unfathomable or ununderstandable.

(21) This, it seems to me, continues to hold even when we move away from the degenerate case of logical truth and consider theories that do indeed satisfy Popper's theorem. The statement $\forall u \forall v(u = v)$, for example, says that there exists exactly one object. In elementary logic it has infinitely many distinct consequences, including for each natural number i greater than 1 a statement, which we call $\neg\omega_i$ (the point of this notation will become clear in **2.10**), to the effect that there do not exist exactly i objects. But it is surely an absurd conceit to maintain that this theory (plainly false as it is) is beyond our full comprehension. Ryle is worth quoting again: 'though people's understanding of the propositions that they use is in this sense [the sense of grasping all logical connexions] imperfect, there is another sense in which their understanding of some of them may be nearly or quite complete' (1945, p. 7; 1971, p. 198). Indeed, if we restrict ourselves to the calculus of elementary logic with identity and no other predicates or relations, we can without too much difficulty give a characterization of all the consequences of $\forall u \forall v(u = v)$ (which does not mean that we can recognise presumptive consequences of great length). It might be thought that considerations of complexity or effectiveness could enable us to draw a useful distinction here. For within elementary logic with identity the theory $\forall u \forall v(u = v)$ is a decidable theory (Tarski, Mostowski, & Robinson 1953, p. 19); there is a mechanical procedure for determining of any statement whether or not it is a consequence of the theory. Decidable theories, it might be conjectured, are understandable (even if we cannot recognise all their consequences), but undecidable theories are not. Percival briefly alludes to Gödel's theorem concerning the incompleteness of consistent recursively axiomatizable theories of arithmetic (§49), from which follows the undecidability of arithmetic, but he does not contrast undecidable theories with decidable ones. But the trouble with this suggestion is that even elementary logic is undecidable (Church), indeed undecidable in the highest degree; a solution to the decision problem is also a solution to the halting problem for Turing machines (this is the gist of the proof of Church's theorem, due to Büchi, presented in Chapter 10 of Boolos & Jeffrey 1974). On the other hand, elementary Euclidean geometry is decidable (Tarski 1948). Should we identify full intelligibility with decidability, we would therefore find ourselves back in the untenable position of saying that the tautology $y \to y$ is beyond our understanding, and having to admit at the same time that Euclidean geometry is not. But nothing in this paragraph should be taken to deny that some more delicate deployment of ideas from recursion theory might provide real illumination of the problem.

2.7 Logical Independence

(22) Whether our theories are fully understandable, and in what sense they have infinite content, are separate and, I have suggested, independent issues. For the remainder of this section I shall content myself with probing further into the latter problem, the technical one. I suspect that what led Popper (and, in his footsteps, Bartley) to think that the issue of the ununderstandability of our theories amounts to more than the incontrovertible psychological fact that we can be surprised by some of their consequences was something like this. The fact that most statements have infinitely many distinct consequences can easily be conflated with the claim that—as we might put it informally—they have infinitely many different things to say; that their contents consist of infinitely many separate nuggets of information, each distinct from and independent of the others. There is, that is to say, an objective sense in which a theory t, even though finitely axiomatizable, may be beyond us.

(23) I would certainly be prepared to consider this as a relevant difference. But unfortunately no such thing has been demonstrated by the proof in **2.2** above. If we look again at that proof, we shall see that although no one element of the sequence $t \vee a, t \vee b, \ldots$ implies any other, any two elements of it are together logically equivalent to t, and therefore together imply all the others. For since a and b are incompatible by hypothesis, we have (using the distributive law):

$$(t \vee a) \wedge (t \vee b) \equiv t \vee (a \wedge b) \equiv t.$$

The only sense, that is, in which the different elements of the sequence $t \vee a, t \vee b, \ldots$ say different things is that no one says the same as another. But taken together, any two say exactly what all the others say. Moreover there is no possibility that a theory t such as the one with which we started should have an equivalent formulation in terms of an infinite sequence of statements each of which is genuinely independent of all the others. For suppose that t were equivalent to the infinite set $\{z_i \mid i \in \mathcal{N}\}$ of independent statements. By the principle of finitude (often called compactness), if t is derivable from this set, as we are assuming, it is derivable from some finite subset, say $\{z_i \mid i < k\}$. But then:

$$\{z_i \mid i < k\} \quad \vdash \quad t \quad \vdash \quad z_k,$$

and the set $\{z_i \mid i \in \mathcal{N}\}$ is not independent after all.

(24) Any independent axiomatization of an unaxiomatizable theory is infinite, and any independent axiomatization of an axiomatizable theory is finite. The difference is that the word 'independent' can be dropped from the first assertion, but not from the second. Unaxiomatizable theories never look finite, but axiomatizable theories sometimes look infinite. My thesis is that this is something of an optical illusion, a logical hologram, an infinite-dimensional Necker hypercube; what is really a single thing is made to assume simultaneously an infinity of different guises. But once we sort out the dependences among them, the finiteness is restored.

2.8 A Result in the Other Direction

(25) Yet the idea that infinite content implies the existence of infinitely many independent nuggets of information can be pressed a little further. Using a corollary stated (but not explicitly proved) by Popper (1966, p. 349), it is possible to establish the somewhat unexpected proposition that, although t is not equivalent to any infinite independent subset, under the conditions already propounded, it does include, within its content, such a set. This shows that there is one sense (though one that I shall claim to be unimportant) in which we can correctly assert that t does have infinitely many different things to say. Those who want to concentrate on the main problem of the paper, and those who can't be bothered with proofs, are once again permitted to move on, either to **2.10** or, if they are desperate, directly to section **3**.

(26) The result of Popper's that we need is this: if the set T of all true statements is not axiomatizable, and t is axiomatizable and false, then its truth content $\mathsf{T} \vee t$ is not axiomatizable. Phrased more generally, this asserts that if Ω is an unaxiomatizable maximal theory that does not imply t, then $\Omega \vee t$ is an unaxiomatizable subtheory of t. It may be shown that if t has infinite content then there must exist such an unaxiomatizable Ω that does not imply t. It follows that $\Omega \vee t$ is not axiomatizable. Now by a theorem of Tarski (proved informally on p. 362 of 1935), every theory is logically equivalent to an independent set, and hence $\Omega \vee t$ must be equivalent to an infinite independent set (for otherwise, as already noted, it would be finitely axiomatizable).

(27) To complete the proof we need to establish the result attributed to Popper, and to establish also that if t has infinite content then at least one unaxiomatizable maximal theory Ω does not imply it.

(28) First suppose that Ω does not imply t. Since it is maximal, it implies $\neg t$. Now if $\Omega \vee t$ were axiomatizable, so would be its conjunction with $\neg t$. But

$$(\Omega \vee t) \wedge \neg t \equiv (\Omega \wedge \neg t) \vee (t \wedge \neg t) \equiv \Omega,$$

meaning that Ω too would be axiomatizable. This proves Popper's corollary.

(29) If t has infinite content then, as noted in **2.3**, $\neg t$ has an infinite number of maximal extensions. That these cannot all be axiomatizable is part of Theorem 8 of (Mostowski 1937; Tarski 1936, p. 370). Here is the simple proof. If a theory X (whether axiomatizable or not) has infinitely many axiomatizable maximal extensions $\{\phi_i \mid i \in \mathcal{N}\}$, then it is consistent with each finite subset of $\{\neg\phi_i \mid i \in \mathcal{N}\}$, and hence (by the principle of finitude) $\mathsf{X} \cup \{\neg\phi_i \mid i \in \mathcal{N}\}$ is consistent. By Lindenbaum's theorem, X has a maximal extension Φ that is not identical with any element of $\{\phi_i \mid i \in \mathcal{N}\}$ (for maximal theories are pairwise incompatible). Thus $\neg t$ has an unaxiomatizable extension Φ, which cannot also be an extension of t.

2.9 An Example (Kepler's Laws)

(30) It may be concluded that when Popper's original assumption holds, so that there exists an infinite set of statements that pairwise are contradictory and individually do not entail t, then t includes amongst its consequences an infinite independent set. A simple and revealing example is provided by Kepler's three laws, augmented by a finite set of initial conditions sufficiently copious for the prediction of the positions of the planets at all future times. It is plain that within the content of this theory we can find an infinite set of statements that, taken in isolation, are logically independent of each other; statements of the positions of Venus at different times suffice for this, since it is only in the presence of universal laws that there is any logical connection between the position of Venus at one time and its position at any other time. But we should not be eager to conclude that there is any important sense in which Kepler's laws say infinitely many distinct things. On the contrary, one of the virtues of logical systematization and axiomatization is that it enables us to replace scattered sets of results by unifying principles. There is something suspiciously retrograde about the claim that Kepler's laws have an infinity of things to say (just as there is something retrograde about the claim that a theory may be replaced by its Ramsey sentence). We must note in any case that to obtain the full force of Kepler's laws (plus initial conditions) we should need to add to the infinite set of predictions concerning Venus (or any other infinite independent subset of the content of Kepler's laws) some statement (or finite set of statements) that renders all but finitely many of those predictions redundant.

2.10 Why Logical Content is Not Atomic

(31) Lying behind the view that scientific theories are infinitely varied in their consequences there is, I suspect, an atomistic view of content: the idea that there exist minimal independent morsels of information from which the contents of all more informative statements and theories are compounded by finite or infinite conjunction. The reader is warned not to be misled by the use of the word 'bit' in information theory to express what sounds like exactly this idea. It is not the same idea. Bits are not minimal, except in the logically insignificant sense of being representable by expressions of minimal length. Indeed, the atomistic thesis as it stands is untenable. For the only possible candidates for the role of atomic contents would be irreducible statements, and no axiomatizable theory with infinite content can be built entirely from irreducibles. This is easily shown. For a set of k irreducibles generates a theory with 2^k distinct consequences; and an axiomatizable theory that is logically equivalent to the conjunction of denumerably many irreducible statements is, by finitude, equivalent to the conjunction of some finite subset of them, so that we return to the previous case. A simple example is supplied by the calculus described in **2.6**, elementary logic with identity as the only relation: for each positive i the statement $\neg\omega_i$ is irreducible, but the conjunction of all these statements yields a theory Ω ('the number of objects is not finite', or 'the universe is infinite') that is not finitely axiomatizable.

(32) The objection may be strengthened by noting that in many calculi there exist no irreducible theories at all. This follows from the result of Mostowski already cited. An example is provided by ordinary classical sentential calculus with denumerably many sentence letters. In such calculi, of course, all non-tautological theories have infinite content.

2.11 A More Model-theoretic Approach

(33) If an atomistic approach of content is possible at all, it will only be, I think, through a move away from logical content to a construe related to what in **2.4** was called informative content. If we identify the content of a theory not with 'the class . . . of statements that it excludes or forbids' but with the class of maximal theories (or, if you like, models or possible worlds) that it excludes or forbids, then most of the difficulties paraded above disappear. Some theories may exclude only finitely many maximal theories; if such theories exist, they will be just the same as those with finite contents—in other words, they will be equivalent to the conjunction of a finite number of irreducibles. But usually, and in some calculi always, non-tautological theories will succeed in excluding infinitely many maximal theories. The main difficulty lies in explaining in what sense maximal theories can be thought of as independent of each other; that is, in showing that if an axiomatizable theory has infinite content this is not simply a result of duplication. Plainly the sense required is not simple logical independence, since any maximal theory is implied by the conjunction of two others. And the hunch that maximal theories can never duplicate each other, or get in each other's way, and that any set of maximal theories can constitute a content, is unfortunately false. For example, in the calculus just mentioned, no theory can exclude the maximal theory Ω unless it also excludes one (in fact, almost all) of the ω_i (this was in effect proved at the end of **2.8** above). Despite these worries, it is quite easy to defend the view that an axiomatizable theory can

make an infinite number of independent exclusions, and this suffices for the claim that it has genuinely infinite content. Whether it entitles us to claim that the theory is infinitely applicable as well is quite a different matter. In section **3** it will be suggested that it does not so entitle us.

2.12 Sceptical Summary

(34) I have been trying without success to find something defensible in the view that the infinitude of a theory's content has more than psychological significance; that there is an objective sense in which it is true that an axiomatizable theory must say more than we can ever appreciate. In his discussion of the syllogism Mill (1843, Book II, Chapter III, section II) rightly dismisses any attempt 'to attach any serious scientific value to such a mere salvo as the distinction drawn between being involved *by implication* in the premises, and being directly asserted in them'. In other words, an axiomatizable theory t does directly (if not transparently) assert in finitely many words everything that its infinitely many consequences take infinitely many words to assert. Mill expresses puzzlement that 'a science, like geometry, *can* be all "wrapt up" in a few definitions and axioms' (*loc. cit.*). But we should not allow ourselves to be taken in here. Although virtually all theories 'wrap up' infinitely many thoughts in the sense that we can find infinitely many thoughts within them, it is a capital mistake to suppose that a theory's content is synthesized from logically more primitive (weaker) components. As we have seen, in many cases there are no weakest components. (Aristotle's treatment of Zeno's paradox of Achilles and the tortoise invites comparison.) I am not of course defending the view that the understanding of a rich scientific theory is a straightforward business, and that some acquaintance with the theory's consequences is not essential to its understanding. As I have already noted, I incline to the view that understanding a theory fundamentally means understanding the problem situation it addresses, and how well it addresses it. It is possible to go further and to recognise that understanding may be enhanced when it is realized that the theory solves, or is unable to solve, some unexpected, some newly emerged problem. I am quite happy to admit that newly identified consequences may lead to a sharp improvement in the understanding of a theory. My purpose here is only to question the doctrine that much can be explained by the infinitude of a theory's content alone. It is such a flimsy matter that we could hardly expect it to yield substantial returns.

3 The Application of Scientific Theories

(35) Even if I am not right about this, I am convinced that the infinite content of our theories is of no great significance when we seek to explain the prodigious and perhaps unquenchable feats of technology. This may seem a blatant platitude, since most of the technical objections made above fizzle out if 'very large but finite content' is substituted for 'infinite content', whereas the attractiveness of the explanation itself is hardly affected. But I believe that the matter goes deeper than that.

(36) The principal consideration here is that theories are not exploited in practice by exploiting their consequences one by one, either routinely or imaginatively, but only in a negative way. What is important about scientific theories is not what they allow (which is a great deal, most of it useless), but what they forbid. The point was made by Popper (1957), §20, and has been well stated by Albert (1968), p 221: 'The foremost function of the nomological science, under practical aspects, is to point out limits of realizability'. I have expatiated on the matter at some length in (Miller 1994, Chapter 2.2*g*, especially pp. 39-41), but Alain Boyer has made me realize that I slightly overstated my case there. I should accordingly like to present the case anew, though more briefly.

(37) The crucial point is indeed stated perfectly well by Percival himself (§52):

> I must make clear at this point that I do not subscribe to the popular view that every technological decision and action (including inventions) is prescribed by one or more scientific theories; in fact none are. This would overlook the fact that scientific laws are universal and therefore can only proscribe; alone, they can tell us only what cannot happen, not what will happen, and therefore alone cannot tell us what we should do to achieve a given end. Building a bridge, car, space-ship and tube of toothpaste is a matter for engineers discovering sets of constructible [I should prefer to say *realizable*] initial conditions that typically lead efficiently to the desired result. This is a conjecture and refutation affair. Universal theories of science help the engineer insofar as they can be used to eliminate some of the hopeful candidates of efficient sets of initial conditions, namely the ones whose description contradicts the accepted scientific theories.

(38) Our theories themselves, that is to say, do not describe positively any event in the world. Only when they are supplemented by initial conditions, do they say anything categorical (to use old fashioned jargon) rather than something that is merely hypothetical. But that is exactly what inventions are: supplementary sets of (reproducible) initial conditions, without which our scientific theories are practically sterile. This is not to deny that scientific knowledge can provide inspiration. Electrodynamics suggests innumerable possibilities for using electric currents to generate motion; but it gives no hint as to how a workable electric razor, for example, may be constructed. Nor is it to deny that the historical sciences (such as cosmogony, geology, evolutionary biology) are concerned with singular statements as much as with universal theories. But these singular statements are not what technology is looking for.

(39) Once we have a proposed set of realizable conditions (invention) in front of us, we may be able to use our scientific knowledge to show that it will not do the job it is designed for. For this purpose the representability of our knowledge as an infinite collection of separate items of information is neither here nor there: the critical deployment of a component of a scientific theory can, objectively & logically speaking, be replaced by the critical deployment of the whole theory. Of course, that is not the only possibility: indeed, our knowledge may imply that the invention will be successful, and even explain why it will be successful. But in each case the invention must be proposed before the theory can be put to work. Inventions are not generated by a positive application of theoretical knowledge, though they may be eliminated by a negative application. It is in this sense that science can tell us only which inventions, which practical proposals, will not work, and cannot tell us which ones will work. As I say, a practical proposal may be explained by science, but it will never be a mere consequence of it.

(40) The Baconian doctrine that science provides the key to the mastery of nature is not rendered less false by being endorsed by Nietzsche in *The Will to Power* ('Science is the transformation of Nature into concepts for the purpose of mastering Nature') or by Habermas. Each confuses science with technology, thus masking the true relationship between the two. As several writers have recently observed (Grove 1989, p. 26; Wolpert 1992, p. 28; Stevenson & Byerly 1995, p. 2), for most of their histories science and technology proceeded independently of each other, and if there was any influence one way or another it was almost always from technology to science. Why should this have been so if science is, as so often thought, the inspiration of technology? The plain answer (not offered by any of the authors mentioned) is that science has nothing to contribute to technology except for criticism; and that this criticism can always in principle be replaced by empirical rather than theoretical criticism; that is, by practical testing. What is characteristic of modern technology is that many practical tests that on grounds of ethics, safety, or cost, simply cannot be performed are replaced by theoretical evaluation, evaluation that takes account of well tested scientific laws. This does not make technology a part of science, let alone an offshoot of science. And as I say, in principle the scientific evaluation is always expendable. Since the purpose of a technological innovation is to be practically successful, here at least the test of practice may be allowed to rule supreme.

(41) To be sure, inductivists and others who imagine that our scientific knowledge is open to justification or to empirical support usually imagine also that this justification seeps down to the inventions whose efficacy our knowledge has the ability to explain. Classical mechanics, inductivists will say, does not only explain the operation of, say, a barometer or a system of pulleys, it also justifies our confidence that such items of machinery will work properly. This surely is a service that science can offer technology beyond the purely eliminative one that I have outlined. But sadly, none of this is true. There exists no process of induction that allows scientific knowledge to attain any positive degree of justification. It is because of this that justificationist anti-inductivists such as Watkins (1984) regard the pragmatic problem of induction (the problem of how science is rationally applied) as the Achilles heel of falsificationism. They are quite wrong to do so. The application of scientific knowledge, like the growth of knowledge itself, consists only of conjectural forays into the unknown and eliminations of failures.

(42) Almost all these points may be made also about the contribution that mathematics makes to technology. It is transparent that a mathematical calculation on its own yields no positive information, and cannot help to solve any practical problem. A calculation—an interpreted one, of course—may in contrast make manifest the shortcomings of a proposed solution; though again a practical test or series of tests might serve as well. It is of some interest that mathematics seems to have been harnessed in this way in technology long before empirical science was.

(43) What is crucial to the growth of technology is that we maintain our level of inventiveness, our ability to conjure up ingeniously wrought assemblages of initial conditions with which to supplement our scientific theories. 'In science we investigate ... reality; in technology we create a reality according to our design' writes Skolimowski (1966, p. 374; quoted by Grove 1989, p. 46). 'Technology, unlike science, is not concerned with things as they are but with things as they might be' says Grove (*loc. cit.*). In other words, the growth of new invention requires a certain openness of the universe to our meddling, rather than the opposite. If the true theory of the cosmos were, as determinists pretend, in need only of a handful of initial conditions in order to have the power to predict all that would happen, then our scope for innovation would be sadly curtailed. Far from its being the case that technological progress is attributable to the infinite content of our scientific theories, such progress is entertainable only because our theories are not too strong. Animals with limited behavioural repertoires—that is, with endosomatically entrenched theories that go a long way to determining their interactions with the rest of the world—have less scope than we have for moulding the world to their desires. Nothing could show more clearly that it is not scientific knowledge that provides an inexhaustible economic resource, but extra-scientific (or perhaps para-scientific) invention.

(44) I cannot quite understand how Percival can have endorsed trenchantly the view that scientific theory has only negative impact on technology, and then at once have proceeded to ignore it. Dare I suggest that he may have moved from the perfectly correct claim that the greater informative content a theory has the more it excludes, through the equally correct claim that the stronger a theory is the more technologically useful it is, to the incorrect conclusion that it must be the inexhaustible content of our best scientific theories, if anything, that is the engine of unlimited technological growth? The argument is invalid because the principal service that science provides to the development of new inventions is a garbage-disposal service. (After the inventions have been invented, science may explain why they work.) Far from promoting technological growth science frustrates it (especially if our theories are false and encourage us to eliminate inventions that would actually function). The faster we clear away mistakes, no doubt, the better, and highly informative theories will be more efficacious in this regard than are weaker ones. But technological success cannot be built on the elimination of errors alone. Weeding a garden, though essential for growth, is not enough to make even a single flower bloom.

4 Conclusion

(45) In these comments I have been concerned not so much to attack Percival's thesis about the possibility of indefinite technological growth as to question the soundness of his argument. This may seem like a thoroughly perverse activity for a criticial rationalist to engage it, since it is one of our principal claims—or anyway, one of mine—that what matters is the truth of an investigator's conclusion, rather than any argument purporting to lead to it. That is indeed so. But Percival himself stresses (§§23-28) the distinction between explaining a state of affairs and justifying the claim that it will be realized, and makes explicit that he is not attempting to provide any kind of justification of the generalization that 'resources and resource-augmenting inventions do not simply dry up' (§26). His goal is only to show that the continued emergence of new inventions, and of new applications of old inventions, which might appear almost miraculous from a crassly materialist point

of view, is explicable. I have tried to say why I think that his explanation is defective, but not by denying the thesis at which it is directed. It is not clear to me how one could attack such a thesis, which states merely a possibility, head on.

(46) In conclusion it would be disingenuous not to acknowledge that, lurking behind the optimistic assertion that interminable economic growth is possible, there is sometimes a decidedly callous doctrine concerning the current availability for exploitation of the material resources to which we have immediate access. Percival says nothing at all about this, and I ascribe to him nothing. But some thinkers will be ready to maintain that, our ingenuity being as untrammelled as it is, we should not be afraid of consuming as vigorously as we like any resources that we can get our hands on. We need not fear the exhaustion of oil supplies, some will say, for as oil becomes scarcer, intellectual investment in the creation of alternative fuels, and of alternative means of transport, will more than compensate for its unavailability. Percival cites some claims by Simon (1981) to the effect that price data show that some minerals have actually become less in demand in the past two centuries, though presumably they have not in any objective sense become less scarce. (Changes of taste, rather than availability of substitutes, may be responsible for some of these falls in price.) To this line of thinking it suffices to say that the guiding principle of liberal social action is that, even if large-scale, it should be piecemeal and, if not reversible—for no innovation is reversible—at least controllable. Social engineering that plans to pay off its mortgage with indefinite future wealth, neither earned nor even properly invested for, fits ill into this liberal picture.

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